

Lösungen zu Logarithmen

Gymnasium

Klasse 9 / 10

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Aufgabe 1:

a) $\log_2 8 = 3$

b) $\log_3 \sqrt{3} = 0,5$

c) $\log_4 \frac{1}{16} = -2$

d) $\log_5 (-25)$ existiert nicht

e) $\log_{\frac{1}{2}} 8 = -3$

f) $\log 0,01 = -2$

g) $\log \sqrt[5]{10^3} = \frac{3}{5}$

h) $\log_4 \sqrt[6]{2} = x \Rightarrow 4^x = 2^{\frac{1}{6}} \Rightarrow 2^{2x} = 2^{\frac{1}{6}} \Rightarrow 2x = \frac{1}{6} \Rightarrow x = \frac{1}{12}$

i) $\log_{\sqrt{2}} \sqrt[3]{2} = x \Rightarrow 2^{\frac{1}{2}x} = 2^{\frac{1}{3}} \Rightarrow x = \frac{2}{3}$

j) $\log(0)$ existiert nicht

Aufgabe 2:

a) $\log_a a = 1$

b) $\log_a a^n = n$

c) $\log_a \sqrt{a^3} = \frac{3}{2}$

d) $\log_a \frac{1}{\sqrt[3]{a^2}} = -\frac{2}{3}$

e) $\log_{\frac{1}{a}} a^2 = -2$

f) $\log_{\frac{1}{a}} \sqrt[3]{a^5} = -\frac{5}{3}$

Aufgabe 3:

a) $\log_2 x = 128 \Rightarrow x = 2^{128}$

b) $\log_5 0,2 = x \Rightarrow 5^x = \frac{1}{5} \Rightarrow x = -1$

c) $\log_x \sqrt{3} = 0,25 \Rightarrow x^{0,25} = \sqrt{3} \Rightarrow \sqrt[4]{x} = \sqrt{3} \Rightarrow x = 9$

d) $\log_{27} x = \frac{2}{3} \Rightarrow x = 27^{\frac{2}{3}} = 9$

e) $\log 10^8 = x \Rightarrow x = 8$

f) $\log(\log x) = 1 \Rightarrow \log(x) = 10 \Rightarrow x = 10^{10}$

g) $\log_4(2x) = 4 \Rightarrow 2x = 4^4 \Rightarrow x = 128$

h) $\log_2(\log_2 x) = 3 \Rightarrow \log_2(x) = 8 \Rightarrow x = 2^8$

$$i) \log_x 2 = 0,5 \Rightarrow x^{0,5} = 2 \Rightarrow x = 4$$

$$j) \log_x 4 = -0,5 \Rightarrow x^{-0,5} = 4 \Rightarrow x^{0,5} = \frac{1}{4} \Rightarrow x = \frac{1}{16}$$

$$k) \log_x(a^n) = 2n \Rightarrow x^{2n} = a^n \Rightarrow x = \sqrt[2n]{a^n} = a^{\frac{n}{2n}} = \sqrt{a}$$

Aufgabe 4:

$$a) \log_a \frac{u \cdot v}{w} = \log_a u + \log_a v - \log_a w$$

$$b) \log_a \frac{a}{b \cdot c} = \log_a a - (\log_a b + \log_a c) = 1 - \log_a b - \log_a c$$

$$c) \log_a(u^3 \cdot v^4) = \log_a u^3 + \log_a v^4 = 3 \cdot \log_a u + 4 \cdot \log_a v$$

$$d) \log_a \left(\sqrt[5]{a^4} \cdot \sqrt[3]{b} \right) = \log_a a^{\frac{4}{5}} + \log_a b^{\frac{1}{3}} = \frac{4}{5} + \frac{1}{3} \log_a b$$

$$e) \log_a \frac{b^2 \cdot c \cdot \sqrt{d}}{e \cdot f^3} = \log_a b^2 + \log_a c + \log_a d^{\frac{1}{2}} - (\log_a e + \log_a f^3) \\ = 2\log_a b + \log_a c + \frac{1}{2}\log_a d - \log_a e - 3\log_a f$$

$$f) \log_a \left(\frac{x^2 \cdot 2y}{\sqrt{z}} \right)^{\frac{1}{3}} = \frac{1}{3} \cdot \left[\log_a x^2 + \log_a 2 + \log_a y - \log_a z^{\frac{1}{2}} \right] \\ = \frac{2}{3}\log_a x + \frac{1}{3}\log_a 2 + \frac{1}{3}\log_a y - \frac{1}{6}\log_a z$$

$$g) \log_a \sqrt{\frac{a}{b}} = \frac{1}{2}\log_a \frac{a}{b} = \frac{1}{2}(\log_a a - \log_a b) = \frac{1}{2} - \frac{1}{2}\log_a b$$

$$h) \log_a \sqrt[3]{\frac{5b^3c^5}{18de^3}} = \frac{1}{2}(\log_a 5 + \log_a b^3 + \log_a c^5 - \log_a 18 - \log_a d - \log_a e^3) \\ = \frac{1}{2}\log_a 5 + \frac{3}{2}\log_a b + \frac{5}{2}\log_a c - \frac{1}{2}\log_a 18 - \frac{1}{2}\log_a d - \frac{3}{2}\log_a e$$

$$i) \log_a \frac{3b+2c}{b-2d} = \log_a(3b+2c) - \log_a(b-2d)$$

$$j) \log_a((x+y)^3 \cdot (x-y)^5) = 3\log_a(x+y) + 5\log_a(x-y)$$

$$k) \log_a \frac{x(x-y)}{(3x+2y)^2} = \log_a x + \log_a(x-y) - 2\log_a(3x+2y)$$

$$l) \log \frac{4ab^3}{2x+y} = \log 4 + \log a + 3\log b - \log(2x+y)$$

$$m) \log((x+y)^2 \cdot a^2 \cdot \sqrt[3]{b}) = 2\log(x+y) + 2\log a + \frac{1}{3}\log b$$

$$n) \log \frac{3\sqrt{x} \cdot \frac{5}{b}}{6yz} = \log 3 + \frac{1}{2} \log x + \log 5 - \log b - \log 6 - \log y - \log z$$

Aufgabe 5:

$$a) \log_3 5 - \log_3 15 + \log_3 \frac{1}{9} = \log_3 \frac{5 \cdot \frac{1}{9}}{15} = \log_3 \frac{1}{27} = -3$$

$$b) 2 \cdot \log_a b - \log_a c = \log_a b^2 - \log_a c = \log_a \frac{b^2}{c}$$

$$c) 2 \cdot \log_b(ab) - \log_b \sqrt{b^3} + \log_b \frac{1}{a^2} = \log_b(ab)^2 - \log_b b^{\frac{3}{2}} + \log_b a^{-2} \\ = \log_b \frac{(ab)^2 \cdot a^{-2}}{b^{1,5}} = \log_b \frac{a^2 b^2 \cdot a^{-2}}{b^{1,5}} \log_b b^{0,5} = 0,5$$

$$d) \log_b(b^2 - 9) - \log_b(b + 3) - \log_b(b - 3) + \log_b \sqrt{b} = \log_b \frac{(b^2 - 9) \cdot \sqrt{b}}{(b + 3)(b - 3)} = \log_b \sqrt{b} = 0,5$$

$$e) \log_3(x + 5) - \log_3(5x + 25) + 2 \cdot \log_3(\sqrt{5x}) = \log_3 \frac{(x + 5) \cdot (\sqrt{5x})^2}{(5x + 25)} = \log_3 \frac{(x + 5) \cdot 5x}{5(x + 5)} = \log_3 x$$

$$f) (\log_2 u^2 - \log_2 u + \log_2 \sqrt{u}) : \log_2 u^3 = (\log_2 \frac{u^2 \cdot \sqrt{u}}{u}) : \log_2 u^3 = \frac{\log_2 u^{1,5}}{\log_2 u^3} = \frac{1,5 \log_2 u}{3 \log_2 u} = 0,5$$

Aufgabe 6:

$$\log(a - b) + \log \sqrt{a + b} - \log \frac{a^2 - b^2}{a^2 + 2ab + b^2} = \log((a - b) \cdot \sqrt{a + b}) - \log \frac{(a + b)(a - b)}{(a + b)^2} \\ = \log \frac{(a - b) \cdot (a + b)^{0,5}}{\frac{a - b}{a + b}} = \log(a + b)^{1,5} = 1,5 \cdot \log(a + b) = 1,5 \cdot 2 = 3$$

Aufgabe 7:

$$a) \log_2 \frac{1}{9} = \log_2 9^{-1} = -1 \cdot \log_2 9 \approx -3,170$$

$$b) \log_2 18 = \log_2(2 \cdot 9) = \log_2 2 + \log_2 9 \approx 1 + 3,170 = 4,170$$

$$c) \log_2 3 = \log_2 \sqrt{9} = \frac{1}{2} \cdot \log_2 9 \approx 1,585$$

$$d) \log_2 2,25 = \log_2 \frac{9}{4} = \log_2 9 - \log_2 4 \approx 3,170 - 2 = 1,170$$

Aufgabe 8:

$$a) \log_4 64 = x \Leftrightarrow 4^x = 64 \Leftrightarrow x = 3$$

$$b) \log_5 \frac{1}{25} = x \Leftrightarrow 5^x = \frac{1}{25} \Leftrightarrow 5^x = 5^{-2} \Leftrightarrow x = -2$$

$$c) \log_5 \frac{1}{\sqrt{5}} = x \Leftrightarrow 5^x = \frac{1}{\sqrt{5}} \Leftrightarrow 5^x = 5^{-\frac{1}{2}} \Leftrightarrow x = -\frac{1}{2}$$

$$d) \log_{10} \sqrt[4]{1000} = x \Leftrightarrow 10^x = \sqrt[4]{1000} \Leftrightarrow 10^x = 1000^{\frac{1}{4}} \Leftrightarrow 10^x = (10^3)^{\frac{1}{4}} \Leftrightarrow 10^x = 10^{\frac{3}{4}} \\ \Leftrightarrow x = \frac{3}{4}$$

Aufgabe 9:

$$a) \log_5 2 = \log_5 \sqrt{4} = \log_5 4^{\frac{1}{2}} = \frac{1}{2} \log_5 4 = \frac{0,8614}{2} = 0,4307$$

$$b) \log_5 20 = \log_5 (4 \cdot 5) = \log_5 4 + \log_5 5 = 0,8614 + 1 = 1,8614$$

Aufgabe 10:

$$a) \log_2 64 = x \Rightarrow 2^x = 64 \Rightarrow x = 6$$

$$b) \log_{25} x = 0,5 \Rightarrow 25^{0,5} = x \Rightarrow x = 5$$

$$c) \log_x 81 = 4 \Rightarrow x^4 = 81 \Rightarrow x = \sqrt[4]{81} = 3$$

$$d) \log_2 6 + \log_2 \frac{8}{3} = x \Rightarrow \log_2 \left(6 \cdot \frac{8}{3} \right) = x \Rightarrow \log_2 16 = x \Rightarrow 2^x = 16 \Rightarrow x = 4$$

$$e) \log_5 125 = x \Rightarrow 5^x = 125 \Rightarrow x = 3$$

$$f) \log_7 \sqrt{343} = x \Rightarrow 7^x = \sqrt{343} \Rightarrow 7^x = \sqrt{7^3} \Rightarrow 7^x = 7^{1,5} \Rightarrow x = 1,5$$

$$g) \log_8 x = \frac{1}{3} \Rightarrow 8^{\frac{1}{3}} = x \Rightarrow x = 2$$

$$h) \log_x 512 = 3 \Rightarrow x^3 = 512 \Rightarrow x = \sqrt[3]{512} \Rightarrow x = 8$$

$$i) \log_2 32 = x \Rightarrow 2^x = 32 \Rightarrow x = 5$$

$$j) \log_x 8 = \frac{3}{2} \Rightarrow x^{\frac{3}{2}} = 8 \Rightarrow x = 8^{\frac{2}{3}} = 4$$

$$k) \log_3 \frac{1}{27} = x \Rightarrow 3^x = \frac{1}{27} \Rightarrow x = -3$$

$$l) \log_x 125 = 3 \Rightarrow x^3 = 125 \Rightarrow x = \sqrt[3]{125} = 5$$

$$m) \log_4 x = 2,5 \Rightarrow 4^{2,5} = x \Rightarrow x = 4^2 \cdot 4^{0,5} = 16 \cdot 2 = 32$$

$$n) x = \log_4 64 \Rightarrow 4^x = 64 \Rightarrow x = 3$$

$$o) \log_{27} x = \frac{2}{3} \Rightarrow 27^{\frac{2}{3}} = x \Rightarrow x = \sqrt[3]{27^2} = 9$$

$$p) x = \log_a (a \cdot \sqrt[3]{a}) = \log_a a + \log_a \sqrt[3]{a} = 1 + \frac{1}{3} \log_a a = 1 + \frac{1}{3} = \frac{4}{3}$$

Aufgabe 11:

$$a) \log_3(3 \cdot \sqrt{2,4}) = \log_3 3 + \log_3 \sqrt{2,4} = 1 + \frac{1}{2} \cdot \log_3 2,4 = 1 + 0,3985 = 1,3985$$

$$b) \log_3 7,2 = \log_3(3 \cdot 2,4) = \log_3 3 + \log_3 2,4 = 1 + 0,7969 = 1,7969$$

Aufgabe 12:

$$\log_a(ab) + \log_a(a^b) + \log_a(b^a) = \log_a a + \log_a b + b \cdot \log_a a + a \cdot \log_a b$$

$$= 1 + 3 + b \cdot 1 + a \cdot 3 = 4 + b + 3a$$

Aufgabe 13:

$$a) \log_a(x^2 - y^2) = \log_a(x+y)(x-y) = \log_a(x+y) + \log_a(x-y) = 4 + 3 = 7$$

$$b) \log_a(x^2 + 2xy + y^2) = \log_a(x+y)^2 = 2 \cdot \log_a(x+y) = 2 \cdot 4 = 8$$

$$c) \log_a(ax + ay) = \log_a a \cdot (x+y) = \log_a a + \log_a(x+y) = 1 + 4 = 5$$

$$d) \log_a \frac{\sqrt{a}}{x-y} = \log_a \sqrt{a} - \log_a(x-y) = 0,5 - 3 = -2,5$$

Aufgabe 14:

$$a) \log_a(uw) = \log_a u + \log_a w = 4 + 5 = 9$$

$$b) \log_a\left(\frac{a}{u}\right) = \log_a a - \log_a u = 1 - 4 = -3$$

Aufgabe 15:

$$a) x = \log_3(3^5) = 5 \cdot \log_3 3 = 5$$

$$b) \log_{\sqrt{3}} 27 = x \Rightarrow \sqrt{3}^x = 27 \Rightarrow (3^{0,5})^x = 3^3 \Rightarrow 3^{0,5x} = 3^3 \Rightarrow 0,5x = 3 \Rightarrow x = 6$$

$$c) \log_2 x = -3 \Rightarrow 2^{-3} = x \Rightarrow x = \frac{1}{8}$$

$$d) \log_x 32 = 5 \Rightarrow x^5 = 32 \Rightarrow x = \sqrt[5]{32} = 2$$

Aufgabe 16:

$$a) x = \log_2 \frac{1}{\sqrt{8}} = \log_2 \sqrt{8}^{-1} = -1 \cdot \log_2 8^{0,5} = -0,5 \cdot \log_2 8 = -0,5 \cdot 3 = -1,5$$

$$b) \log_8 x = \frac{1}{3} \Rightarrow 8^{\frac{1}{3}} = x \Rightarrow x = \sqrt[3]{8} = 2$$

$$c) \log_8 x = -\frac{1}{3} \Rightarrow 8^{-\frac{1}{3}} = x \Rightarrow x = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$$

$$d) \lg(3x + 10) = 2 \Rightarrow 10^2 = 3x + 10 \Rightarrow 3x = 90 \Rightarrow x = 30$$

$$e) \log_2 x = 9 \Leftrightarrow 2^9 = x \Leftrightarrow x = 512$$

f) $\log_x \frac{1}{9} = 4 \Leftrightarrow x^4 = \frac{1}{9} \Rightarrow x = \sqrt[4]{\frac{1}{9}} = \sqrt{\frac{1}{3}}$ (die negative Lösung dieser Gleichung entfällt, da die Basis des Logarithmus nicht negativ sein kann)

g) $\log_{10}(\log_{10} x) = 0 \Leftrightarrow 10^0 = \log_{10} x \Leftrightarrow 1 = \log_{10} x \Leftrightarrow 10^1 = x \Leftrightarrow x = 10$

Aufgabe 17:

$$\log_a 6 + \log_a 21a - \log_a 63 + \log_a \sqrt{a} = \log_a \frac{6 \cdot 21a \cdot \sqrt{a}}{63} = \log_a (2 \cdot a^{1.5}) = \log_a 2 + \log_a a^{1.5} \\ = \log_a 2 + 1.5$$

Aufgabe 18:

$$\log(a-b) + \log \sqrt{a+b} - \log \frac{a^2 - b^2}{a^2 + 2ab + b^2} = \log \frac{(a-b) \cdot \sqrt{a+b}}{a^2 - b^2} = \log \frac{(a-b) \cdot \sqrt{a+b} \cdot (a^2 + 2ab + b^2)}{a^2 - b^2} \\ = \log \frac{(a-b) \cdot \sqrt{a+b} \cdot (a+b)^2}{(a+b)(a-b)} = \log(a+b)^{1.5} = 1.5 \cdot \log(a+b) = 1.5 \cdot 2 = 3$$

Aufgabe 19:

$$\log(a^2 - 1) - \log(a-1) - \log((a+1)^2) = \log \frac{a^2 - 1}{(a-1) \cdot (a+1)^2} = \log \frac{(a+1)(a-1)}{(a-1)(a+1)^2} = \log \frac{1}{a+1} \\ = \log(a+1)^{-1} = -\log(a+1)$$

Aufgabe 20:

$$(\log_2 u^2 - \log_2 u + \log_2 \sqrt{u}) : \log_2 u^3 = \log_2 \frac{u^2 \cdot \sqrt{u}}{u} : \log_2 u^3 = \log_2 u^{1.5} : \log_2 u^3 \\ = \frac{1.5 \cdot \log_2 u}{3 \cdot \log_2 u} = \frac{1}{2}$$

Aufgabe 21:

$$\log(x^3 - 4x) + \log \frac{1}{x-2} - \log(x^2 + 2x) = \log \frac{(x^3 - 4x) \cdot \frac{1}{x-2}}{x^2 + 2x} = \log \frac{x \cdot (x^2 - 4) \cdot \frac{1}{x-2}}{x \cdot (x+2)} \\ = \log \frac{(x+2)(x-2) \cdot \frac{1}{x-2}}{x+2} = \log 1 = 0$$

Aufgabe 22:

$$2\log_3(2\sqrt{a}) - \log_3(7a) + \log_3 \frac{7}{4} = \log_3(2\sqrt{a})^2 - \log_3(7a) + \log_3 \frac{7}{4} = \log_3 \frac{4a}{7a} \cdot \frac{7}{4} = \log_3 1 = 0$$